Heat transfer model of superheated vapor bubbling in liquid in multi-stage refrigeration systems

Authors: Lorenzo CARRIERI, Gianpiero COLANGELO, Giuseppe STARACE
Dept. of Engineering for Innovation, University of Salento, LECCE, 73100, Italy

Presenter: prof. Giuseppe STARACE
giuseppe.starace@unisalento.it

ICR 2019
THE 25th IIR INTERNATIONAL CONGRESS OF REFRIGERATION
August 24-30 | Montréal, Québec, Canada

Hosted by IIF IIR

Manuscript ID: 502; DOI: 10.18462/iir.icr.2019.502

UNIVERSITÀ DEL SALENTO

THE INTER-COOLING TECHNIQUE BY BUBBLING

In multi-stage (i.e: 2-stage) intercooled compression techniques the low temperature heat sink is obtained internally at an intermediate pressure and is made of the saturated refrigerant expanded after exiting the condenser.
In each desuperheating stage, the temperature of the vapor tends to the saturation value, through a heat transfer process, carried out either by injecting liquid into the discharge line of the low-pressure compressor or by bubbling the vapor into the liquid.

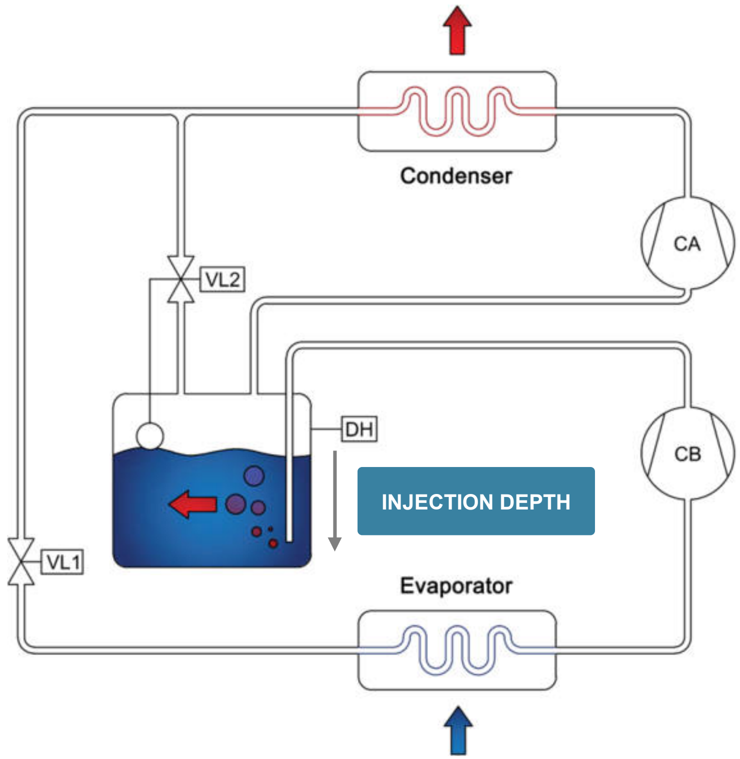
The inter-refrigeration technique by bubbling allows the desuperheating of the steam down to the saturation conditions. **The main design parameter is the injection depth which has to guarantee a sufficient process time and a minimum back-pressure at the low pressure compressor discharge line.**

ADVANTAGES

- Extremely simple change to cycle configuration
- Easy management and adjustment
- High reliability
- Exiting vapor reaches saturation conditions

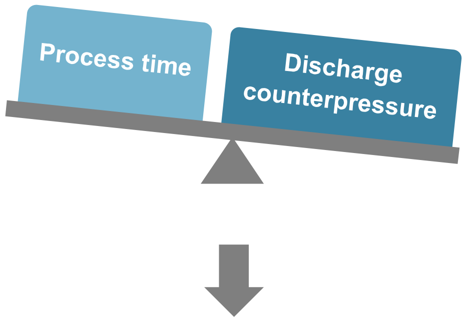
DISADVANTAGES

- Counterpressure at the discharge line of the low pressure compressor
- Low awareness of the mechanisms of heat transfer between bubbles and liquid
- Not optimal dimensioning but choices based on experience



2-STAGE UNIT WITH INTER-COOLED COMPRESSION BY BUBBLING

CB	Low pressure compressor
CA	High pressure compressor
VL1	Expansion valve
VL2	Level adjustment valve
DH	Desuperheater by bubbling



HEAT TRANSFER MODEL INSIDE THE DESUPERHEATER

L. Carrieri, G. Colangelo, G. Starace - **Heat transfer model of superheated vapor bubbling in liquid in multi-stage refrigeration systems**
Dept. of Engineering for Innovation, University of Salento, LECCE, 73100, Italy

MODEL HYPOTHESES

BUBBLE GEOMETRY

- Spherical bubble of time variable radius r_{bub}

BATH PROPERTIES

- Isothermal liquid at the desuperheater saturation temperature
- Liquid bath pressure variable with the depth (*Stevino's law*)
- Non-negligible effects by liquid surface tension (*Laplace's equation*)

FLOW REGIME

- Flow regime with independent bubbles
- Vertical bubble motion driven by bouyancy and hydrodynamical forces

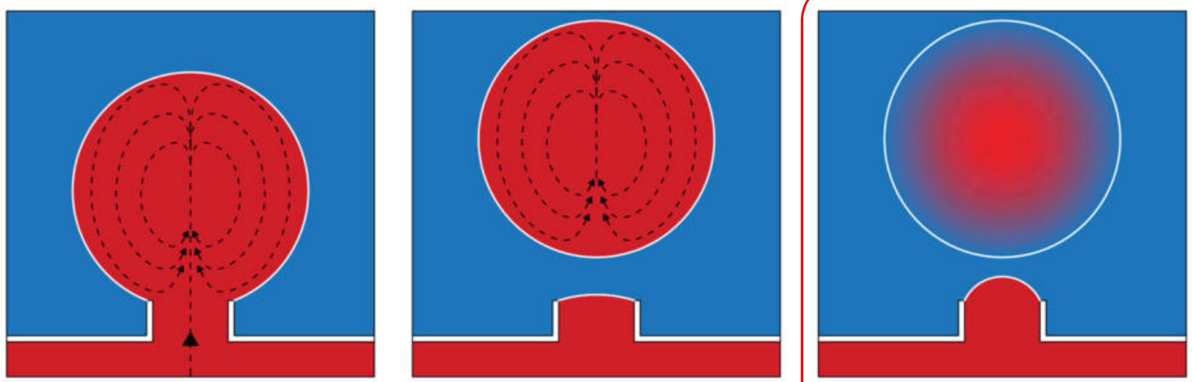
HEAT TRANSFER

- Conduction heat transfer inside the bubble
- The convective heat transfer coefficient calculated according to the following relations

$$\mathrm{Nu} = 0{,}228\mathrm{Ra}^{0{,}226} \qquad \mathrm{Ra} = \frac{g\beta(T_1 - T_2)r_{bub}^3}{\nu^2}\mathrm{Pr}$$

As existing models (bubbles made of saturated vapor and with a relative motion between liquid and vapor only due to radial components of growth and collapse of bubbles) were found not to be applicable to the case of this study (vapor is injected in superheated conditions moving up in the liquid bath driven by Bouyancy force with influence on convection), a new model was developed to be a basis to help designers to dimension desuperheaters.

A semi-analytical single bubble model was proposed starting from the Fourier equation inside the bubble and adapting it to the real boundary conditions of the heat transfer process and including the energy exchanges effects due to both sensible and latent heat.



Filling phase
The superheated vapor is injected in the liquid bath and forms the bubble. Inside the bubble the streamlines are toroidal.
The vapor temperature is uniform

Detachment phase
The toroidal motion continues by inertia, but is soon dissipated by viscosity of the superheated vapor. The temperature is still uniform but slightly lower than the initial one

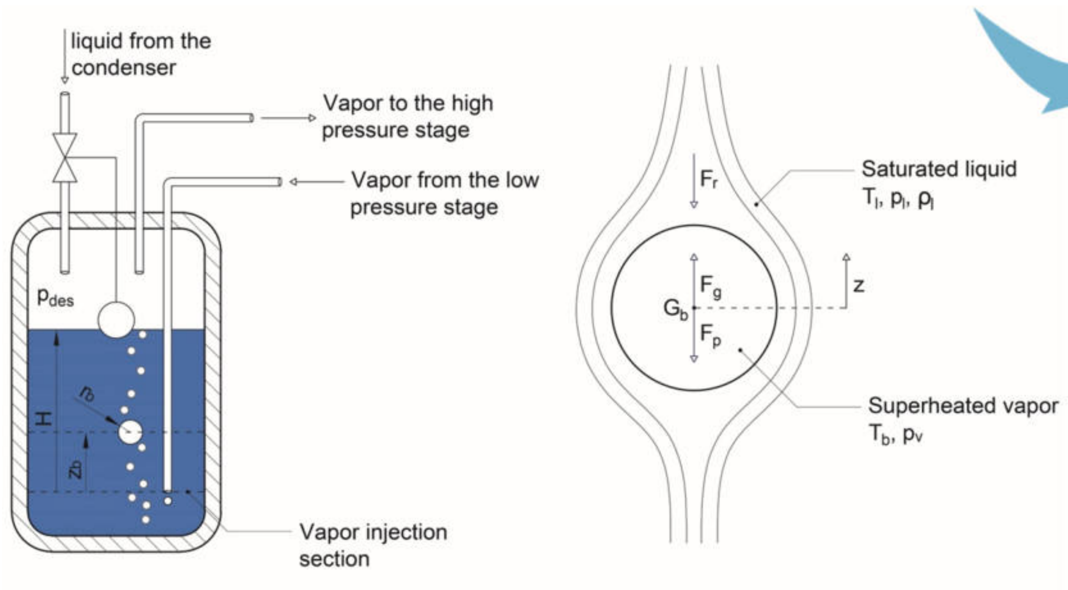
Cooling phase
In absence of convection caused by gravity, the toroidal motion stops and the cooling of the superheated vapor by conduction starts.

L. Carrieri, G. Colangelo, G. Starace - **Heat transfer model of superheated vapor bubbling in liquid in multi-stage refrigeration systems**
Dept. of Engineering for Innovation, University of Salento, LECCE, 73100, Italy

THE BUBBLE MOTION IN THE LIQUID BATH

BUBBLE VERTICAL EQUILIBRIUM

$$\underbrace{\boxed{\bar{\rho}_{bub}\ddot{z}_{bub}}}_{\text{inertial term}} + \underbrace{\boxed{\frac{3\,\mathrm{sgn}(\dot{z})}{8r_{bub}} C_D \rho_l \dot{z}_{bub}^2}}_{\text{hydrodynamic force}} - \underbrace{\boxed{g(\rho_l - \bar{\rho}_{bub})}}_{\text{Bouyancy net force}} = 0$$



Solutions of the non linear differential equation in the time interval of infinitesimal duration, in such a way to neglect coefficient variations

Riccati Equation

CASO 1: $\dot{z}_{bub}(t) > 0$

$$z_{bub}(t) = -\frac{1}{G}\ln\left\{\frac{1}{2}\left[e^{\sqrt{|FG|}t - Gz_0}\left(1 - \frac{\dot{z}_0 G}{\sqrt{|FG|}}\right) + e^{-\sqrt{|FG|}t - Gz_0}\left(1 + \frac{\dot{z}_0 G}{\sqrt{|FG|}}\right)\right]\right\}$$

$$\dot{z}_{bub}(t) = -\frac{\sqrt{|FG|}e^{-\sqrt{|FG|}t - G(z_0 + z_{bub}(t))}}{2G}\left[e^{2\sqrt{|FG|}t}\left(1 - \frac{\dot{z}_0 G}{\sqrt{|FG|}}\right) - \left(1 + \frac{\dot{z}_0 G}{\sqrt{|FG|}}\right)\right]$$

CASO 2: $\dot{z}_{bub}(t) \leq 0$

$$z_{bub}(t) = z_0 - \frac{1}{G}\ln\left[\cos\left(\sqrt{|FG|}t\right) - \frac{G\dot{z}_0}{\sqrt{|FG|}}\sin\left(\sqrt{|FG|}t\right)\right]$$

$$\dot{z}_{bub}(t) = -\frac{1}{G}\left[\frac{-G\dot{z}_0\cos\left(\sqrt{|FG|}t\right) - \sqrt{|FG|}\sin\left(\sqrt{|FG|}t\right)}{\cos\left(\sqrt{|FG|}t\right) - \frac{G\dot{z}_0}{\sqrt{|FG|}}\sin\left(\sqrt{|FG|}t\right)}\right]$$

$$F = \frac{g(\rho_l - \bar{\rho}_{bub,j})}{\bar{\rho}_{bub,j}}$$

$$G = -\frac{3\,\mathrm{sgn}(y)\,C_{D,j}\rho_l}{8r_{bub,j}\bar{\rho}_{bub,j}}$$

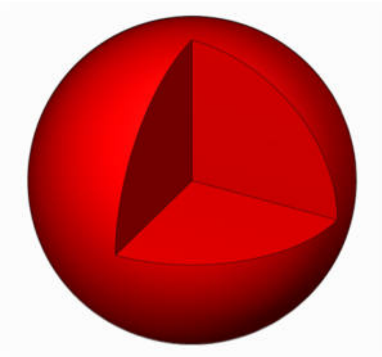
$C_D(\mathrm{Re})$	*hydrodynamic drag coefficient in the time interval* $\Delta\tau_j$
$z_{bub}(t)$	*bubble vertical coordinate with respect to the injection point*
$\rho_l(t)$	*liquid density*
$\bar{\rho}_{bub}(t)$	*mean density of the vapor in the time interval* $\Delta\tau_j$

L. Carrieri, G. Colangelo, G. Starace - **Heat transfer model of superheated vapor bubbling in liquid in multi-stage refrigeration systems**
Dept. of Engineering for Innovation, University of Salento, LECCE, 73100, Italy

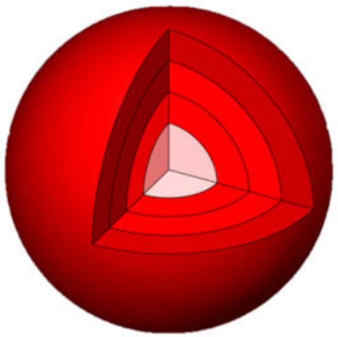
ICR 2019 THE 25th IIR INTERNATIONAL CONGRESS OF REFRIGERATION August 24-30 | Montréal, Québec, Canada

THERMAL CONDUCTION IN THE VAPOR BUBBLE

The cooling process is modelled like a series of isobaric heat exchanges, at constant radius and of infinitesimal duration with an isothermal liquid field and at constant Nusselt number at the liquid–vapor interface. This leads to an **unsteady 1-D heat transfer model with distributed parameters**.



Bubble model
«as a whole»



Bubble model
«with shells»

BUBBLE THERMAL EQUATION

$$\underbrace{\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)}}_{\text{Conduction term}} = \underbrace{\boxed{\frac{1}{\alpha(T)}\frac{\partial T}{\partial t}}}_{\text{Thermal energy variation term}}$$

Non-linear equation as the thermal diffusivity α depends on temperature and so on position and time

Discretizing the vapor properties with respect to the position for infinitesimal time intervals

$$\begin{cases} \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial T_1}{\partial r}\right) = \dfrac{\rho_1 c_{p1}}{\lambda_1}\dfrac{\partial T_1}{\partial t} = \dfrac{1}{\alpha_1}\dfrac{\partial T_1}{\partial t} & 0 \le r \le r_1 \\ \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial T_2}{\partial r}\right) = \dfrac{\rho_2 c_{p2}}{\lambda_2}\dfrac{\partial T_2}{\partial t} = \dfrac{1}{\alpha_2}\dfrac{\partial T_2}{\partial t} & r_1 \le r \le r_2 \\ \vdots & \\ \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial T_M}{\partial r}\right) = \dfrac{\rho_M c_{pM}}{\lambda_M}\dfrac{\partial T_M}{\partial t} = \dfrac{1}{\alpha_M}\dfrac{\partial T_M}{\partial t} & r_{M-1} \le r \le r_M \end{cases}$$

Thermal balance in the vapor domain

r	radial coordinate with respect to the sphere center
$T(r,t)$	thermal radial field in the sphere
$\alpha(T)$	vapor thermal diffusivity

To find the solution the following conditions have to be given

- $2M$ boundary conditions (valid $\forall t$)
- M initial conditions (valid $\forall r \in [r_{k-1}, r_k]$ with $k = 1, \dots, M$)

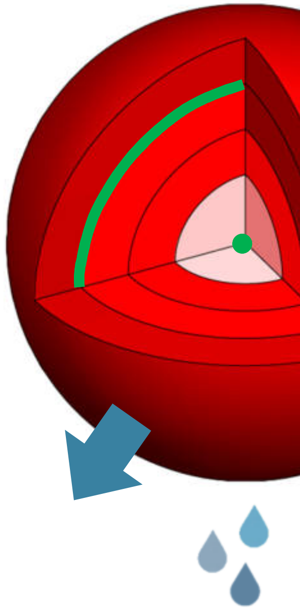
L. Carrieri, G. Colangelo, G. Starace - **Heat transfer model of superheated vapor bubbling in liquid in multi-stage refrigeration systems**
Dept. of Engineering for Innovation, University of Salento, LECCE, 73100, Italy

THERMAL CONDUCTION IN THE VAPOR BUBBLE:
BOUNDARY CONDITIONS WITH THE POSSIBILITY OF PHASE CHANGE DURING COOLING PROCESS

«INTERNAL» BOUNDARY CONDITIONS

$$\begin{cases} \left.\dfrac{\partial T_1}{\partial r}\right|_{r=0} = 0 & \\ \lambda_k \left.\dfrac{\partial T_k}{\partial r}\right|_{r=r_k} = \lambda_{k+1} \left.\dfrac{\partial T_{k+1}}{\partial r}\right|_{r=r_k} & \forall k = 1, \dots M-1 \\ T_k(r_k, t) = T_{k+1}(r_k, t) & \forall k = 1, \dots M-1 \end{cases}$$

- Symmetry of the thermal field
- Consistency of the thermal fluxes at the interface (M - 1 conditions)
- Consistency of the temperatures at the interface between shells (M - 1 conditions)



«EXTERNAL» BOUNDARY CONDITIONS

$$\lambda_M \left.\frac{\partial T_M}{\partial r}\right|_{r=r_M} = -h_{conv}[T(r_M, t) - T_l]$$

$$T_M(r,t)\big|_{r=r_M} = T_{sat}$$

- **Sensible heat transfer**

  Consistency between conductive and convective fulxes on liquid – vapor interface

- **Latent heat transfer**

  Constant surface temperature equal to the saturation value

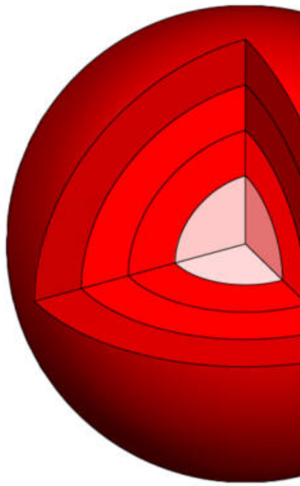
THERMAL CONDUCTION IN THE VAPOR BUBBLE: SOLUTION

$$
\begin{cases}
T(r,t) = \bigcup_{k=1}^{M} T_k(r,t) \\[2ex]
T_k(r,t) = \Omega + \dfrac{1}{r}\displaystyle\sum_{n=1}^{+\infty} c_n \Psi_{kn}(r) e^{-\beta_n^2 t} \\[2ex]
c_n = \dfrac{\sum_{k=1}^{M} \int_{r_{k-1}}^{r_k} w_k r [T_{ik}(r) - T_l] \Psi_{kn}(r) \mathrm{d}r}{\sum_{k=1}^{M} \int_{r_{k-1}}^{r_k} w_k \Psi_{kn}^2(r)\, \mathrm{d}r} \\[2ex]
\Psi_{kn}(r) = A_{kn} \cos\left(\sqrt{\dfrac{\beta_n^2}{\alpha_k}} r\right) + B_{kn} \sin\left(\sqrt{\dfrac{\beta_n^2}{\alpha_k}} r\right)
\end{cases}
$$

Considering uniform thermodynamic properties in the vapor region ($M = 1$)

Solution to the Liouville Problem formulation

SENSIBILE Heat transfer	$T(r,t) = T_l + \dfrac{1}{r}\displaystyle\sum_{n=1}^{\infty} \dfrac{4\beta_n \int_0^{r_{bub}} r[T_i(r) - T_l] \sin(\beta_n r)\, \mathrm{d}r}{2\beta_n r_{bub} - \sin(2\beta_n r_{bub})} \sin(\beta_n r)\, e^{-\alpha\beta^2 t}$
LATENT Heat transfer	$T(r,t) = T_{sat} + \dfrac{1}{r}\displaystyle\sum_{n=1}^{\infty} \dfrac{2}{r_{bub}} \sin(\beta_n r)\, e^{-\alpha\beta^2 t} \int_0^{r_{bub}} r[T_i(r) - T_{sat}] \sin(\beta_n r)\, \mathrm{d}r$



$T_k(r,t)$	thermal field in the k*th* shell
Ω	characteristic temperature of the heat transfer process
c_n	combination constants of the eigenfunctions (dependent on initial conditions)
$\Psi_{kn}(r)$	system eigenfunctions (solutions of the Sturm – Liouville formal problem)
β_n	system eigenvalues

MODEL IMPLEMENTATION

Input parameters BUBBLING DEVICE

- Desuperheating pressure
- Injection depth

Input parameters VAPOR INJECTION

- Bubble initial radius
- Injection temperature
- Injection velocity

Input parameters CALCULATION

- Discretization of the bubble (shell number)
- Number of terms of the solution for the sensible/latent case
- Minimum and maximum timestep

OUTPUT

EXIT CONDITIONS

Bubble reaching surface,
Bubble completely condensed
Bubble collapse for instant condensation

1) Vertical motion analysis

Position and velocity of the bubble in the bath

2) Parameters of the heat transfer

Vapor pressure
Saturation temperature
Nusselt number

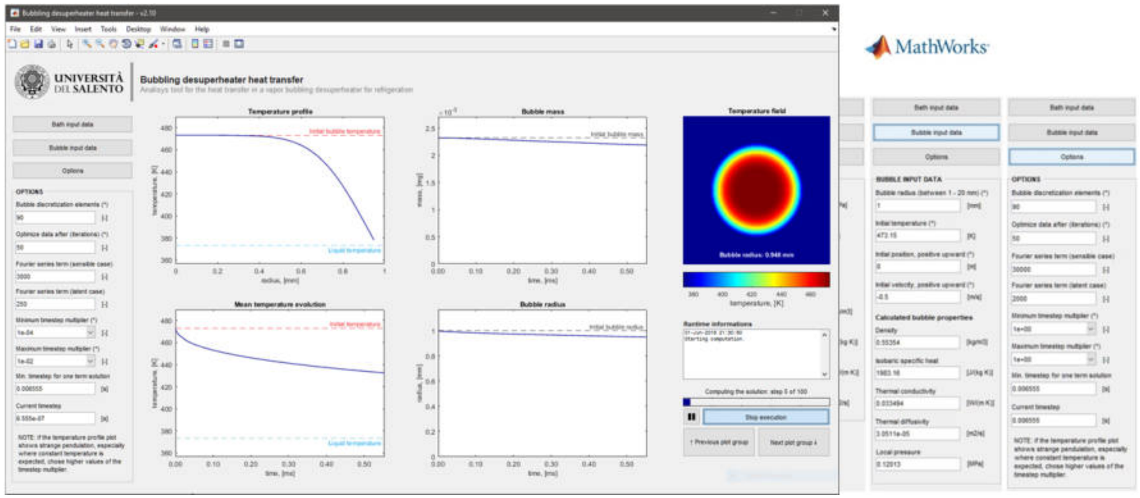
3) Solution of the heat transfer problems

Thermal field
Energy transferred
Mass variation

4) Updating properties and geometry

Bubble density
External radius

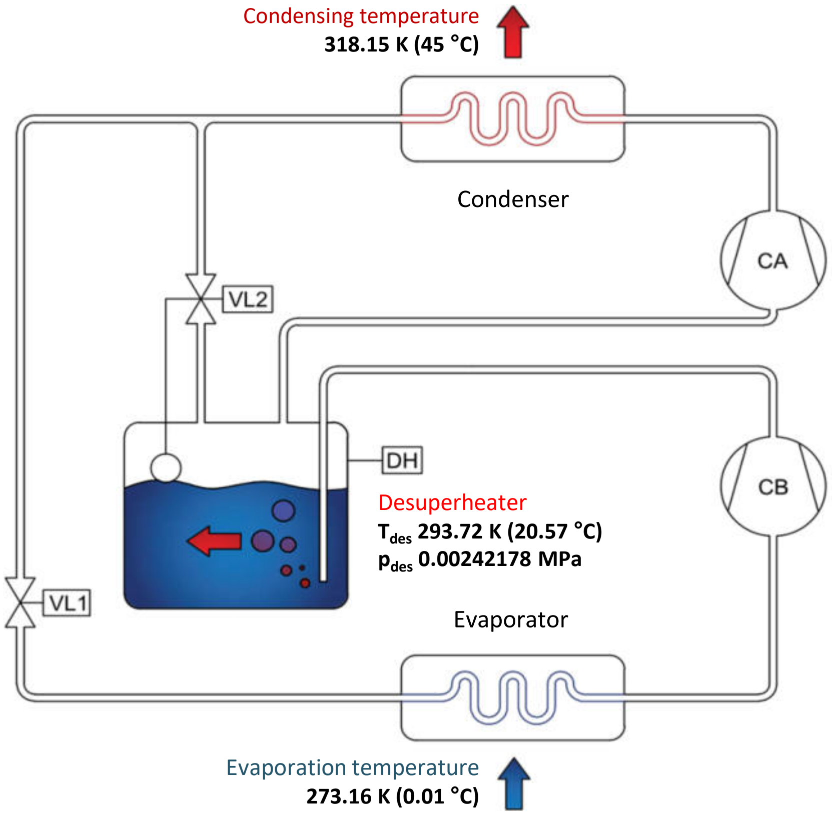
HEAT TRANSFER MODEL VAPOR BUBBLE/LIQUID BATH



L. Carrieri, G. Colangelo, G. Starace - **Heat transfer model of superheated vapor bubbling in liquid in multi-stage refrigeration systems**
Dept. of Engineering for Innovation, University of Salento, LECCE, 73100, Italy

ICR 2019
THE 25th IIR INTERNATIONAL CONGRESS OF REFRIGERATION
August 24-30 | Montréal, Québec, Canada
Hosted by

CASE STUDY: 2-stage WATER cycle between 0.01°C and 45°C



THERMAL MODEL
WITH 5 DEGREES OF FREEDOM

- Desuperheating pressure
- Injection depth
- Vapor injection temperature
- Injection velocity
- Initial bubble radius

Known at optimal conditions at given cycle temperatures

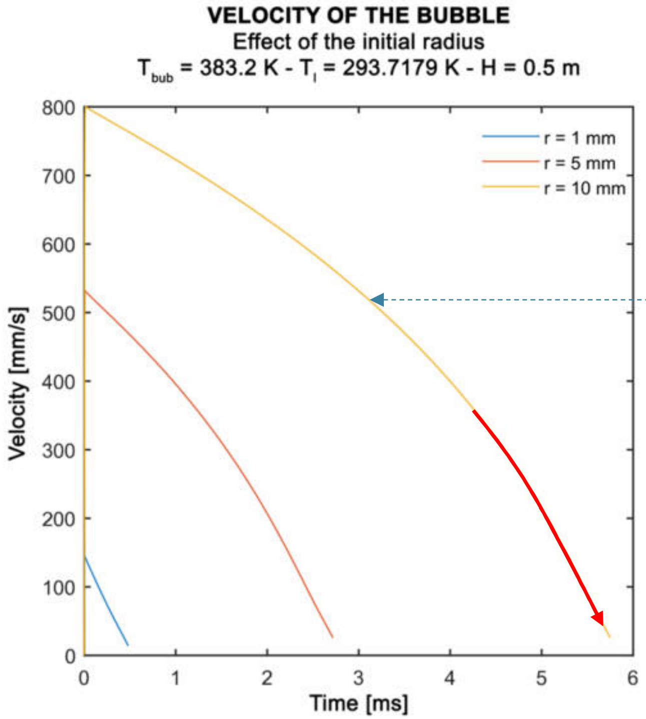
Depends on the nozzle characteristics

Table 1. Parameters of the simulation

Parameter	Value
Desuperheating pressure, p_{des} [MPa]	2.42 x 10^{-3}
Initial velocity, $\dot{z}_0$ [m/s]	0.0
Initial position, z_0 [m]	0.0
Initial radius, r_{b0} [mm]	{1.0; 5.0; 10.0}
Injection temperature, T_{b0} [K]	{383.2; 423.2}
Injection depth, H [m]	{0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7}

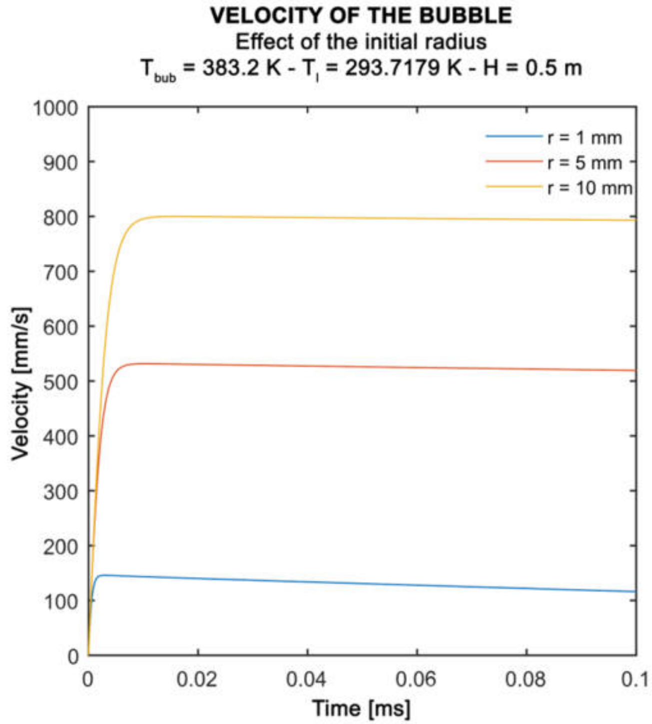
RESULTS: BUBBLE VELOCITY

Bubble velocity and position diagrams when varying the initial radius; the effect of the injection depth and of the injection temperature is not qualitatevely significant. Trends can be explained with reference to the modelled phenomena.



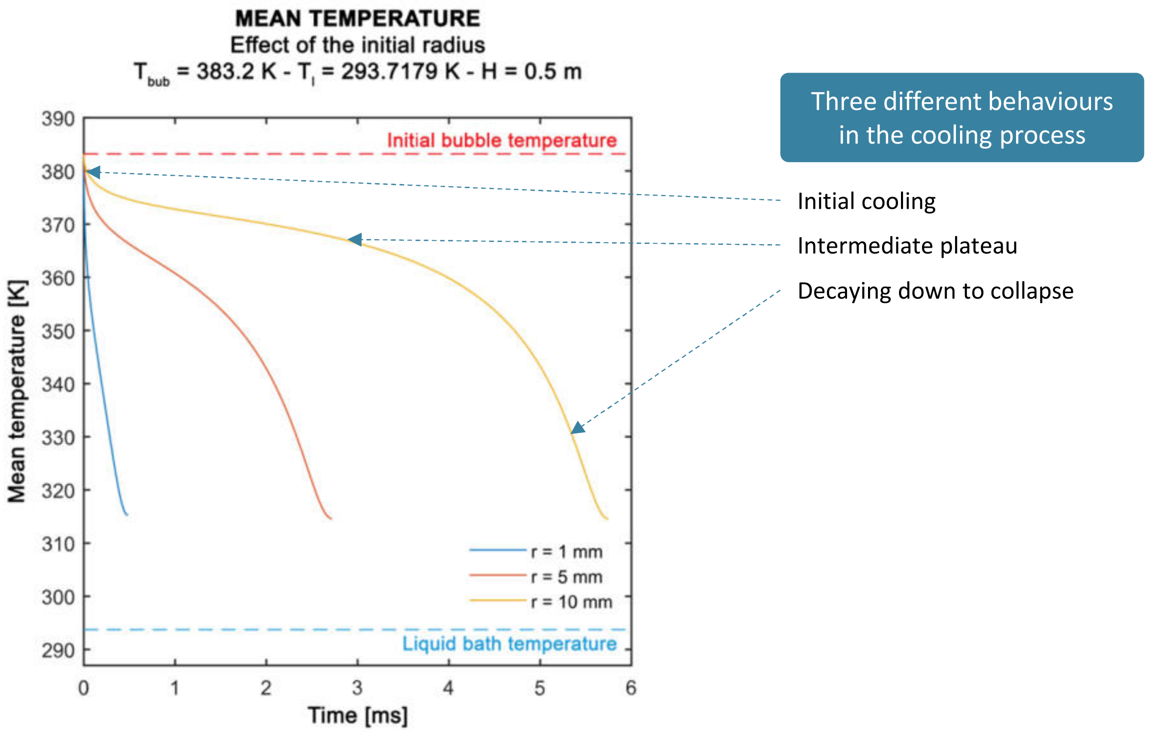
The cooling process is important to the bubble radius and on the vapor density : **There is NOT a final velocity**

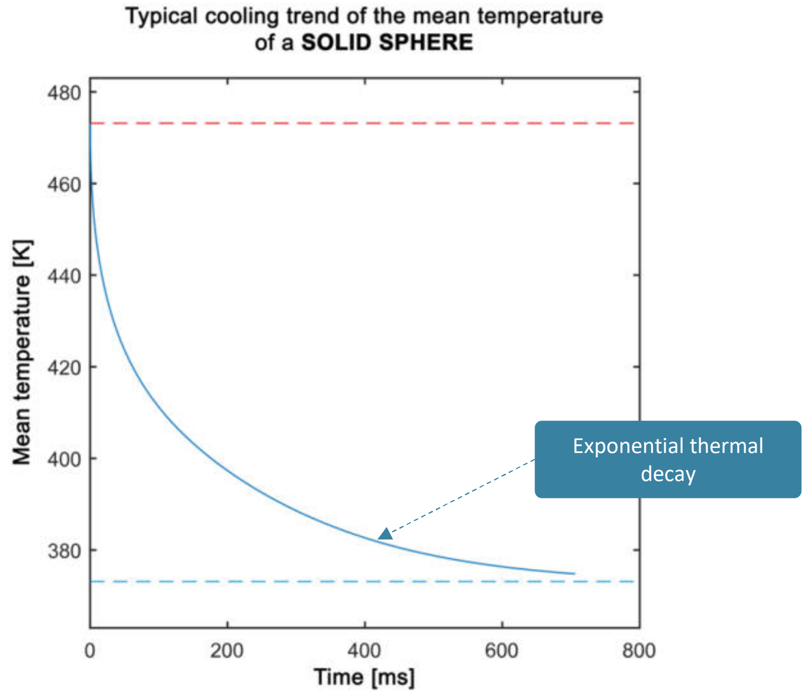
The bubble slows down to stop as its mean density reaches that of the liquid



RESULTS: BUBBLE TEMPERATURE

Bubble mean temperatures and the temperature field show interesting results as they are clearly distinct from those of a solid sphere being uniformly cooled.

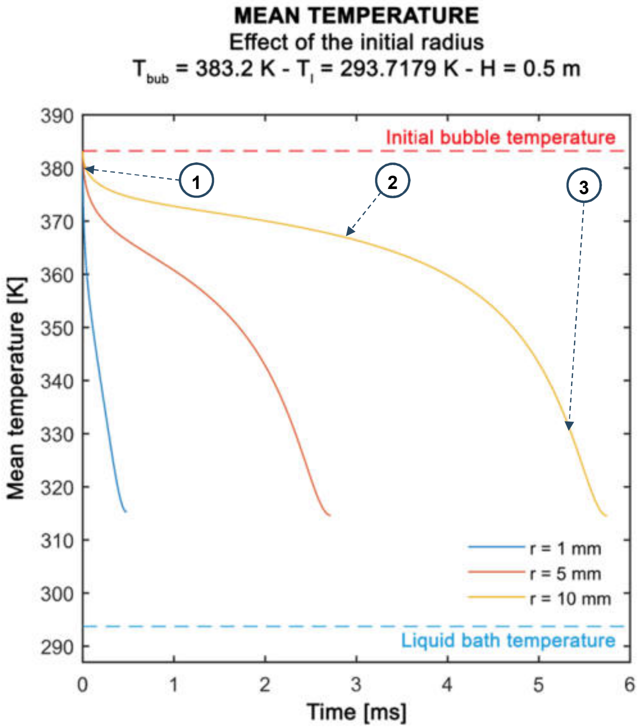


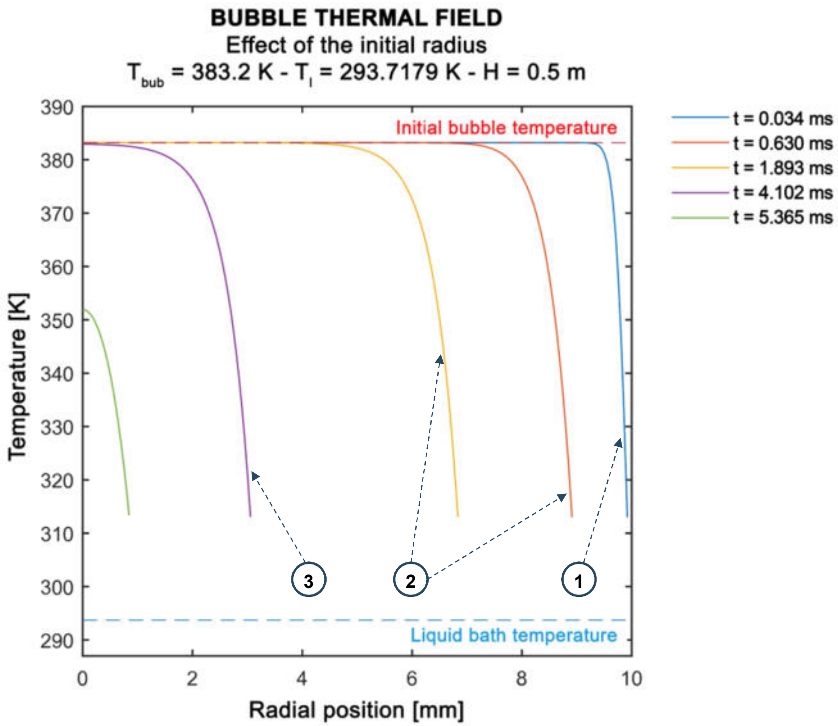


L. Carrieri, G. Colangelo, G. Starace - **Heat transfer model of superheated vapor bubbling in liquid in multi-stage refrigeration systems**
Dept. of Engineering for Innovation, University of Salento, LECCE, 73100, Italy

RESULTS: BUBBLE TEMPERATURE

To explain, it is useful the spatial analysis of the temperature.



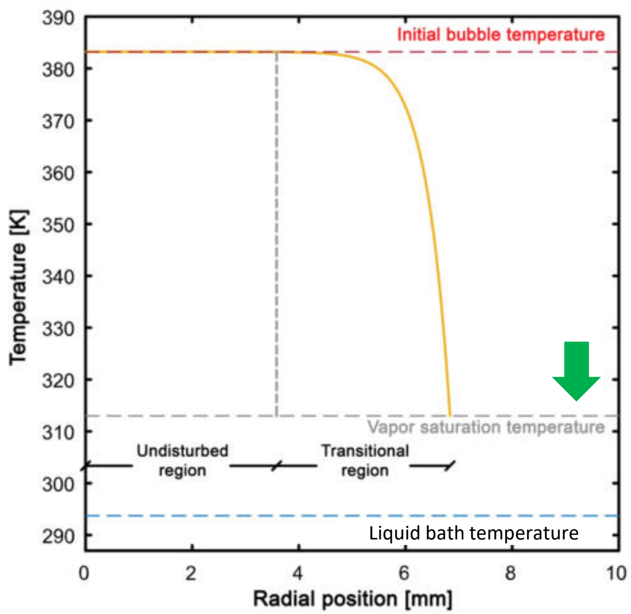


1. Initial cooling
2. Intermediate plateau
3. Decaying down to collapse

RESULTS: BUBBLE TEMPERATURE

Identifying two regions is always possible:

1. the external one, close to the liquid that is influenced by its presence. Here the heat transfer is intense and variations in temperature are evident (**Transitional region**).
2. the internal one that tends to remain at the initial temperature (**Undisturbed region**).



TRANSITIONAL REGION WIDTH:

The «distance from bubble surface in which the vapor temperature varies more than 1% of the initial value»

Vapor injection occurs at pressure higher than p_{des}

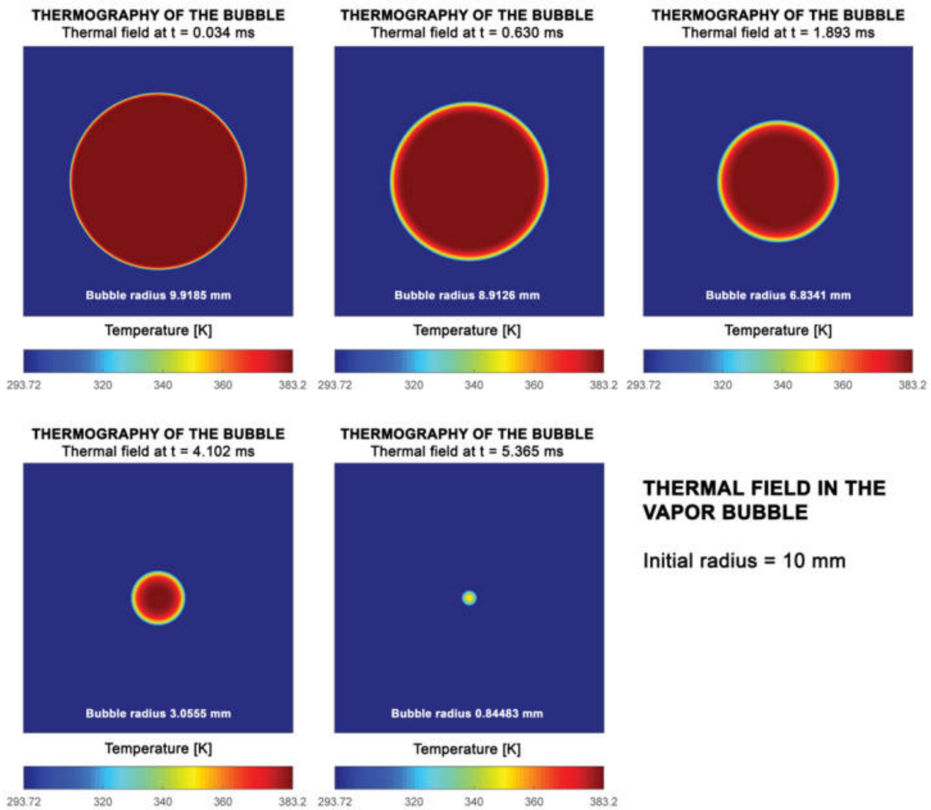
+

Vapor pressure is influenced by the liquid surface tension ($p_v > p_l$)

=

The vapor saturation temperature is higher than the liquid temperature

Vapor condensing on the bubble interface «freezes» the ΔT with the bath **increasing convection and accelerating the cooling process**



THERMAL FIELD IN THE VAPOR BUBBLE

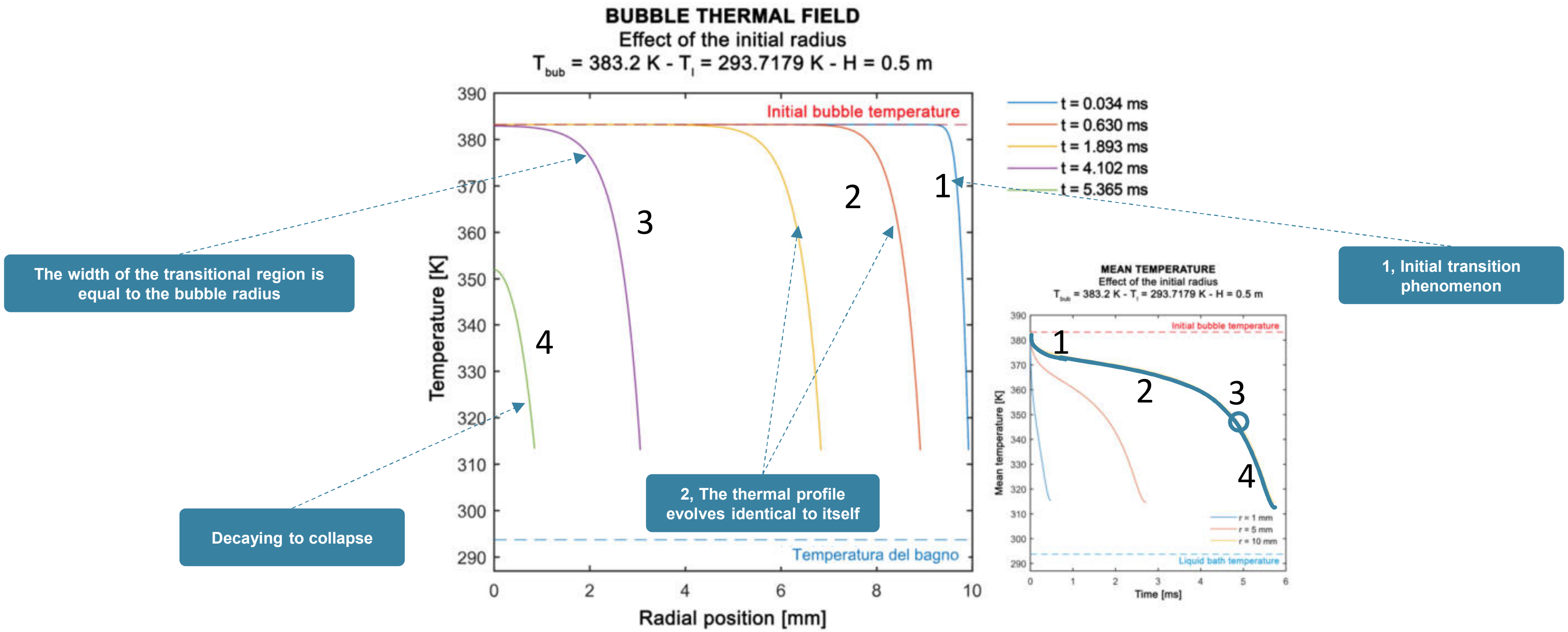
Initial radius = 10 mm

L. Carrieri, G. Colangelo, G. Starace - **Heat transfer model of superheated vapor bubbling in liquid in multi-stage refrigeration systems**
Dept. of Engineering for Innovation, University of Salento, LECCE, 73100, Italy

RESULTS: BUBBLE TEMPERATURE

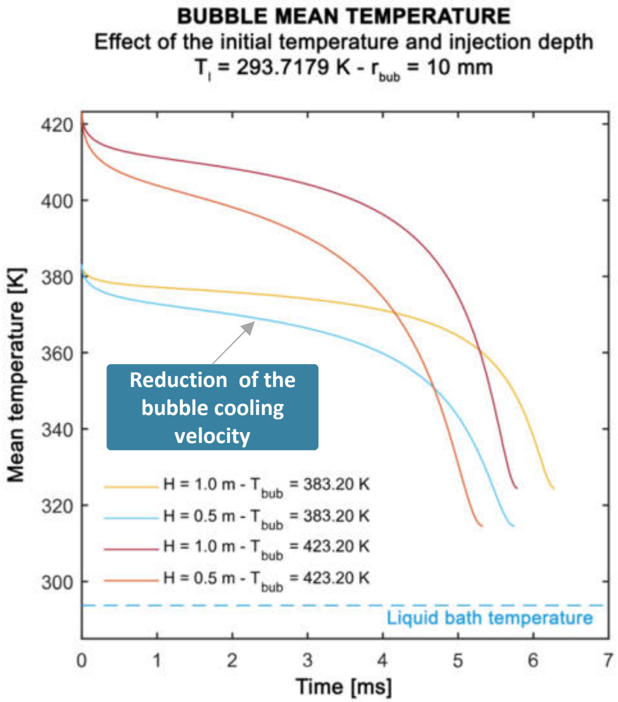
Identifying two regions is always possible:

1. the external one, close to the liquid that is influenced by its presence. Here the heat transfer is intense and variations in temperature are evident (**Transitional region**).
2. the internal one that tends to remain at the initial temperature (**Undisturbed region**).



RESULTS: DISCUSSION

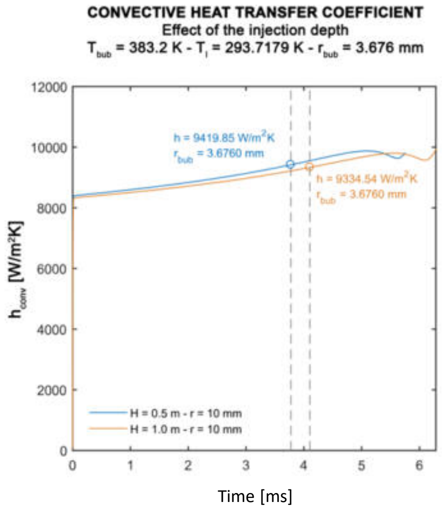
The comparison between the mean temperature obtained for a bubble of the same initial radius varying the injection depth shows some trends that need explanation.

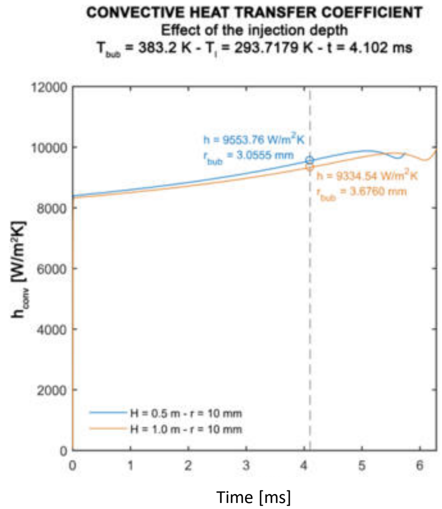


Increasing the injection depth the ΔT between saturation points vapor / liquid increases

The convection heat transfer coefficient does not vary significantly with depth

Even increasing convection towards the liquid, the **cooling velocity of the bubble decreases.**





RESULTS: DISCUSSION

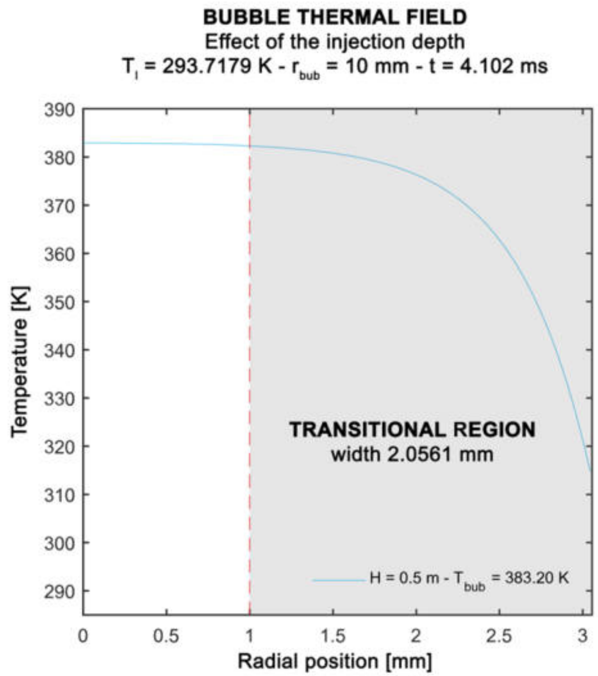
The comparison between the mean temperature obtained for a bubble of the same initial radius varying the injection depth shows some trends that need explanation.

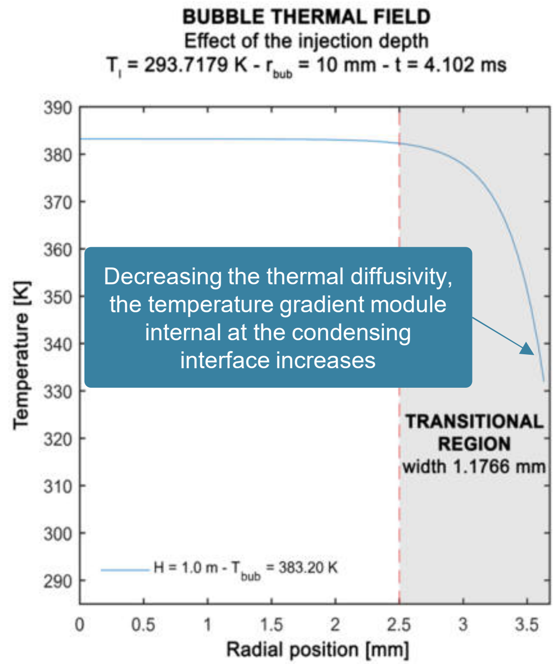
Heat trasferred by **conduction from the adjacent shell**	**Saturation shell**	Heat trasferred by convection to the liquid bath

$$\dot{m}_{l,j} = \frac{4\pi r_{bub,j}^2}{h_{vl}}\left[\bar{k}_{bub}\left.\frac{\mathrm{d}T}{\mathrm{d}r}\right|_{r=r_{bub,j}} + h_{conv}(T_{sat}-T_l)\right]$$

EQUATION OF THE CONDENSING FLOW RATE

The energy totally lost by the bubble (proportional to the condensed mass) does NOT vary significantly.

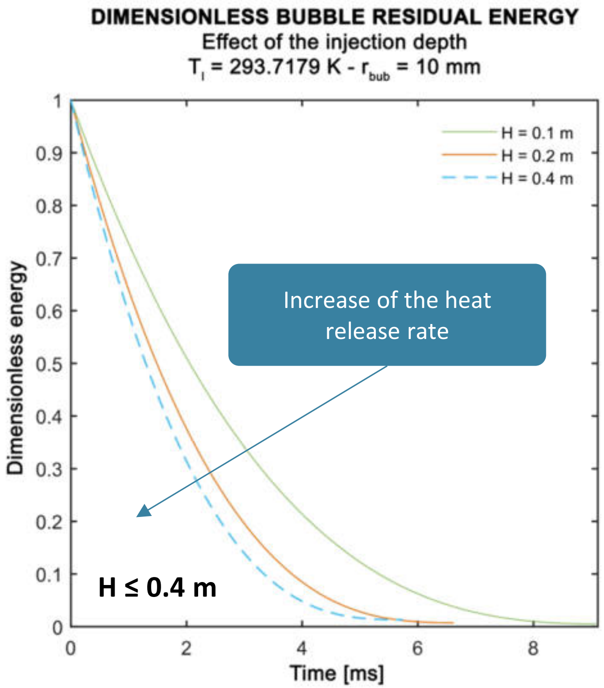


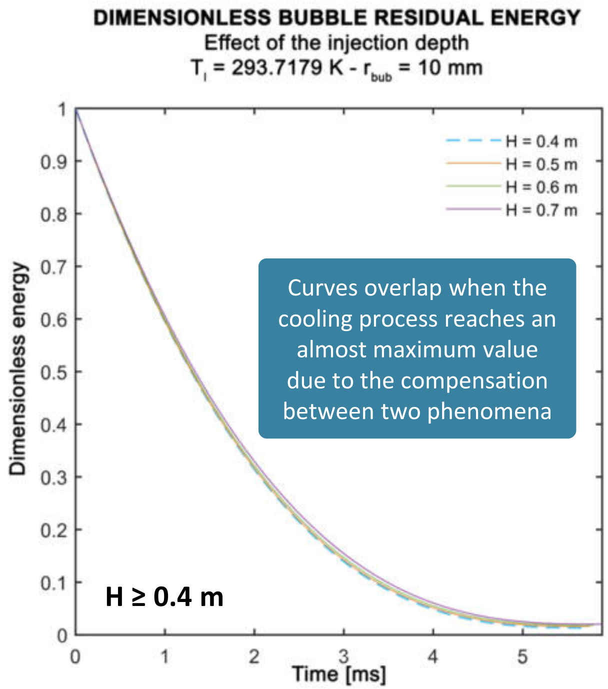


L. Carrieri, G. Colangelo, G. Starace - **Heat transfer model of superheated vapor bubbling in liquid in multi-stage refrigeration systems**
Dept. of Engineering for Innovation, University of Salento, LECCE, 73100, Italy

RESULTS: DISCUSSION

The increase due to the injection depth increases the convective heat transfer towards the liquid but its effect on the total energy release is smoothed by the change in the thermal diffusivity.





DIMENSIONLESS BUBBLE RESIDUAL ENERGY

«ratio between the bubble residual and the total energy transferable to the liquid starting from the the initial saturation condition and ending to the final ones»

$$E^*_{bub}(t) = \frac{\Delta E_{bub,max} - E_{loss}(t)}{\Delta E_{bub,max}}$$

An optimal value of the injection depth exists **that maximizes the energy release ratio**.
Increasing the depth is not convenient as the heat release rate causes only a higher counterpressure at the discharge line of the low stage compressor.

CONCLUSIONS

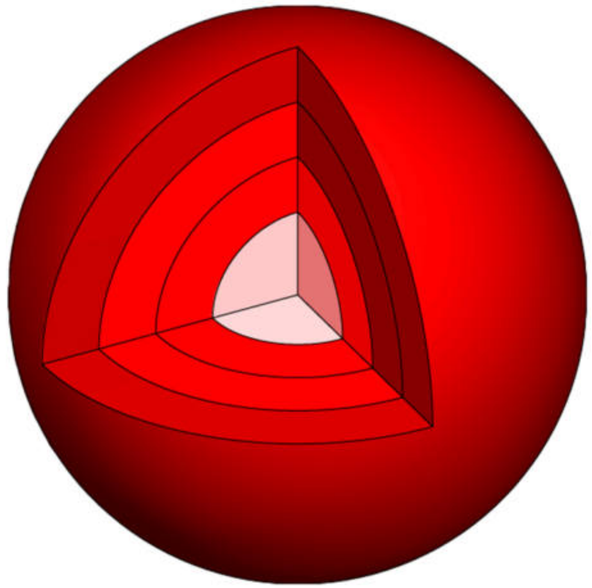
- A semi-analytical model of heat transfer between liquid and vapor has been proposed, using a single bubble approach, adapting the Fourier thermal equation to real cooling conditions and including the effects of sensible and latent heat.

- Model appears to be physically consistent and well interpretable trends can be observed; the liquid surface tension, the injection depth and the thermal diffusivity of the vapor are investigated as antagonistic phenomena in the cooling process.

- Their effect results in the existence of an optimal injection depth, that optimizes the heat transfer, at given thermodynamic conditions of the vapor entering the desuperheater and of the bubble size.

- The model can be proposed as a valid tool to help the design of multi-stage systems

- Further theoretical and experimental investigations are needed for a full validation of the model and for extension to the general case of multiple bubbles flow.



UNIVERSITÀ DEL SALENTO

Thanks for your kind attention

For further info and questions to the presenter and authors:
giuseppe.starace@unisalento.it